North **Lanarkshire** Council

HERIOT WATT UNIVERSITY

ACADÉMIE DE STRASBOURG

Mathematiques Sans Frontieres
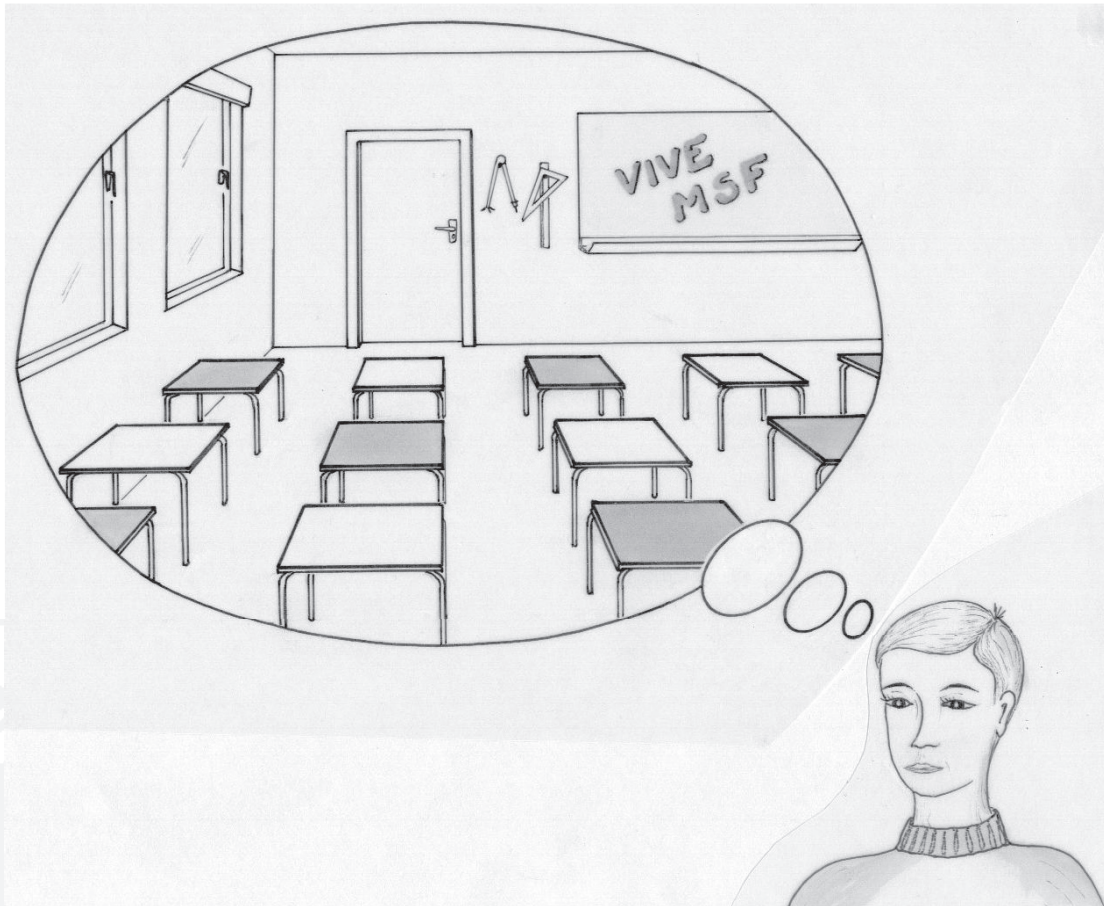
2005

February 24th 2005

You must explain your answers for all the questions except 2, 4, 6 and 7.
Every attempt gets some marks
Careful work is taken into account

Question 1 Langue Vivante

Give your answer in German, French, Spanish or Italian using a minimum of 30 words.

Changez de place

Dans une classe, il y a 5 rangées de 5 tables individuelles. Le professeur demande à ses 25 élèves de changer de place en respectant la consigne suivante : chacun prendra soit la place devant ou derrière celle qu'il occupait, soit celle à sa droite ou à sa gauche.

Pierre sait que son professeur aime plaisanter. Il imagine que les tables sont alternativement de 2 couleurs, comme les cases d'un damier....

« Ce que vous nous demandez est impossible ! » s'écrie-t-il alors , « et je peux vous le prouver. »

Ecrire le raisonnement de Pierre qui démontre l'impossibilité d'un tel mouvement.

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Question 1 Foreign language question

In einem Klassenzimmer stehen in 5 Reihen jeweils 5 Einzeltische. Der Lehrer möchte, dass seine 25 Schüler die Plätze tauschen, indem sich jeder entweder auf den Platz davor, dahinter, rechts oder links setzt.
Peter weiss, dass sein Lehrer die Schüler gerne reinlegt. Peter stellt sich die Tische wie ein Schachbrett vor, abwechselnd weiss und schwarz.
„Was Sie verlangen, geht gar nicht!" ruft er plötzlich. „Und ich kann es Ihnen beweisen!"

Schreibt die Begründung von Peter auf, die zeigt, dass ein solches Vorhaben unmöglich ist.

En una clase, hay 5 filas de 5 mesas individuales. El profesor pide a sus 25 alumnos que cambien de sitio respetando la consigna siguiente : cada uno tendrá que ir o delante, o detrás, o a la izquierda o a la derecha de donde estaba sentado.
Pedro sabe que a su profesor le gusta bromear. Imagina que las mesas son alternativamente de 2 colores como las casillas de un tablero...
"¡ Lo que Usted nos pide es imposible!" dice Pedro "se lo voy a demostrar".

Escribe el razonamiento de Pedro quien demuestra la imposibilidad de tal movimiento.

In una classe ci sono 5 file ciascuna con 5 tavoli. Il professore chiede ai suoi 25 studenti di spostarsi seguendo l'indicazione: "ognuno si sieda davanti o dietro o a destra o a sinistra del posto che sta occupando".
Piero sa che il prof scherza volentieri. Immagina che i tavoli siano alternativamente di due colori, come nella scacchiera.
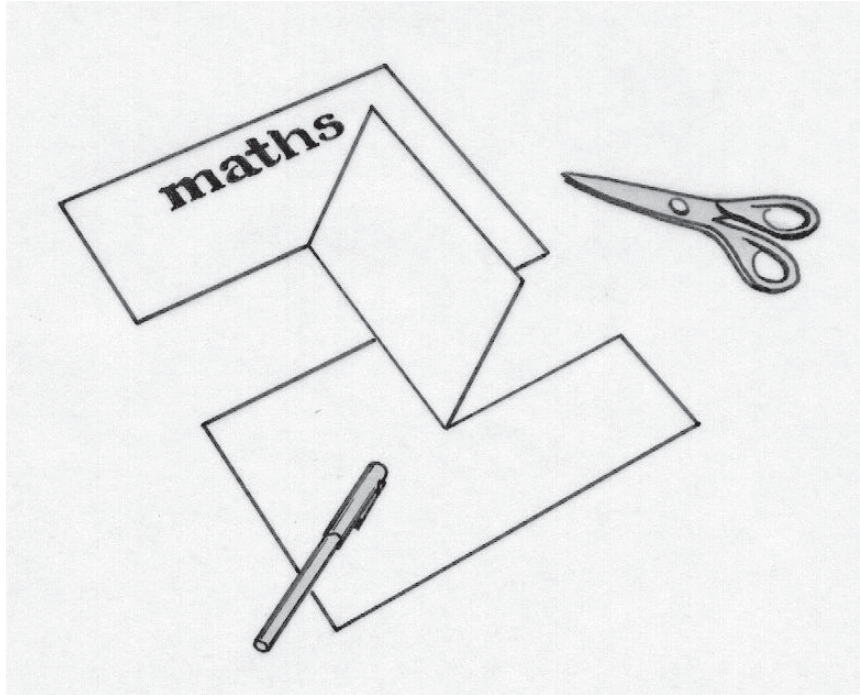"Ciò che ci chiede è impossibile" replica "ed io posso provarlo".

Riprodurre il ragionamento per mezzo del quale Piero riesce a dimostrare l impossibilità di un tale movimento.

Question 2 Cut and no paste

On Anne Marie's desk, Michael finds a sheet of paper that has been cut and folded in a strange way. No glue has been used. Look at this figure.

Cut your answer sheet and then fold it the same way. Be careful to keep your paper in one piece.
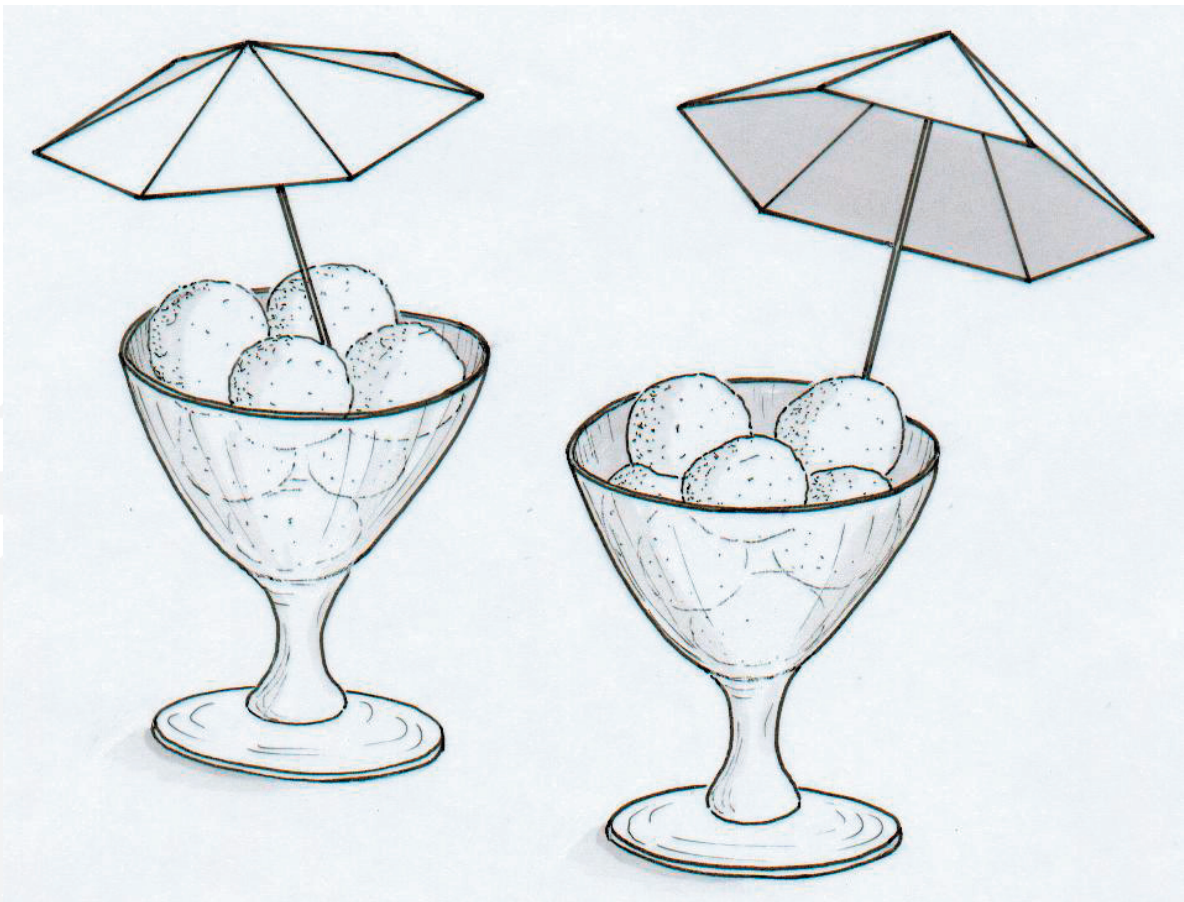
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Question 3 Umbrella.net

To decorate a birthday treat, Icare makes paper umbrellas in the following way :

The umbrellas are in the shape of a pyramid with a regular hexagonal base of side 5 cm. The 6 isosceles triangles forming the pyramid are identical. The edges from the apex to the base are 6 cm long.
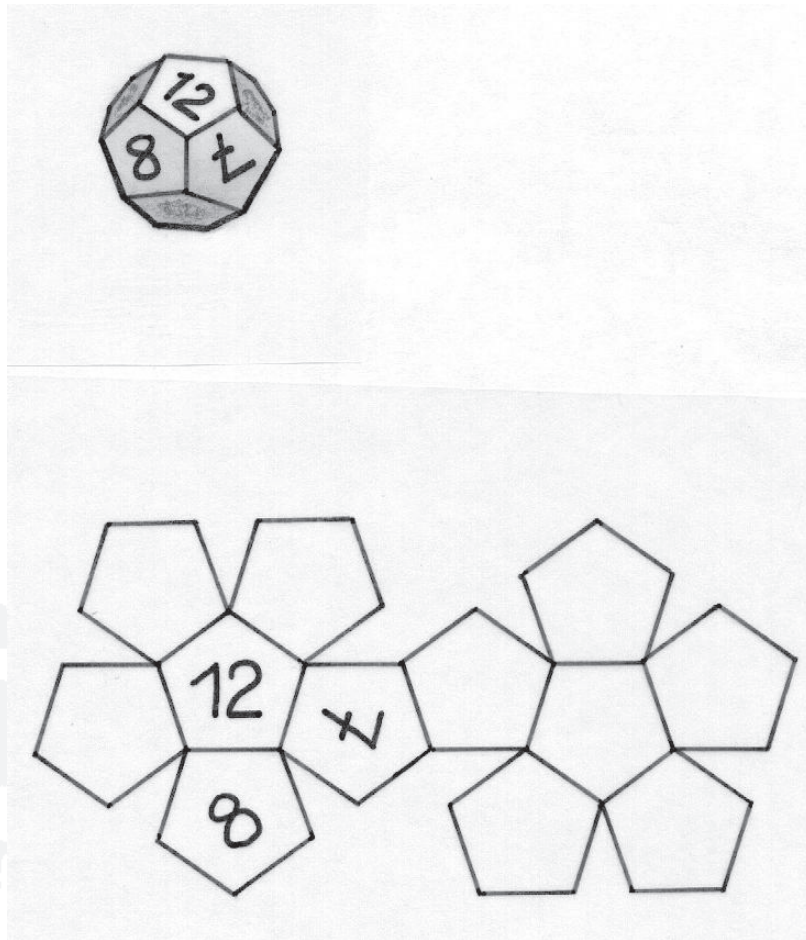Draw the net of this pyramid on a single piece of paper and stick it to your answer sheet.
Work out the height of the pyramid to the nearest millimetre.
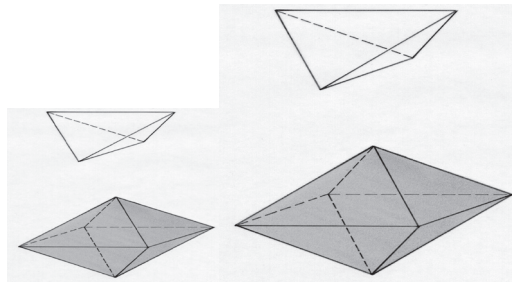
Question 4 Dai's die

Dai, who is a big fan of board games, has a fantastic collection of dice. One of them is in the shape of a dodecahedron. This die has 12 faces, all regular pentagons, with opposite faces parallel, and they are numbered from 1 to 12. In the same way as a six-sided die the sum of the numbers on opposite faces is a constant.
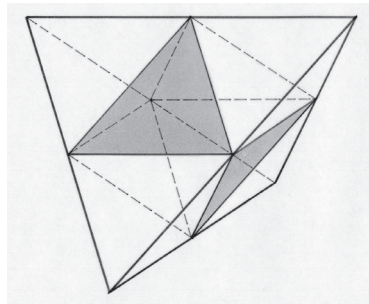Draw the net of a die like this and number its faces.

Note : The singular of dice is die. The Welsh name Dai is pronounced as - die .

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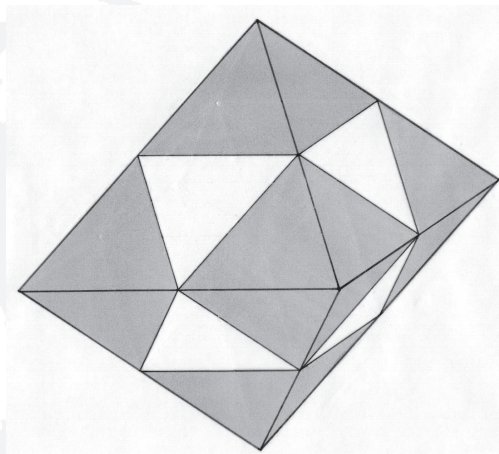
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The diagram above shows a tetrahedron T_1 and an octahedron O_1. All the faces are equilateral triangles of side 1.

Above you can see how to construct a tetrahedron T_2 with side 2 units using tetrahedrons T_1 and a single octahedron O_1.
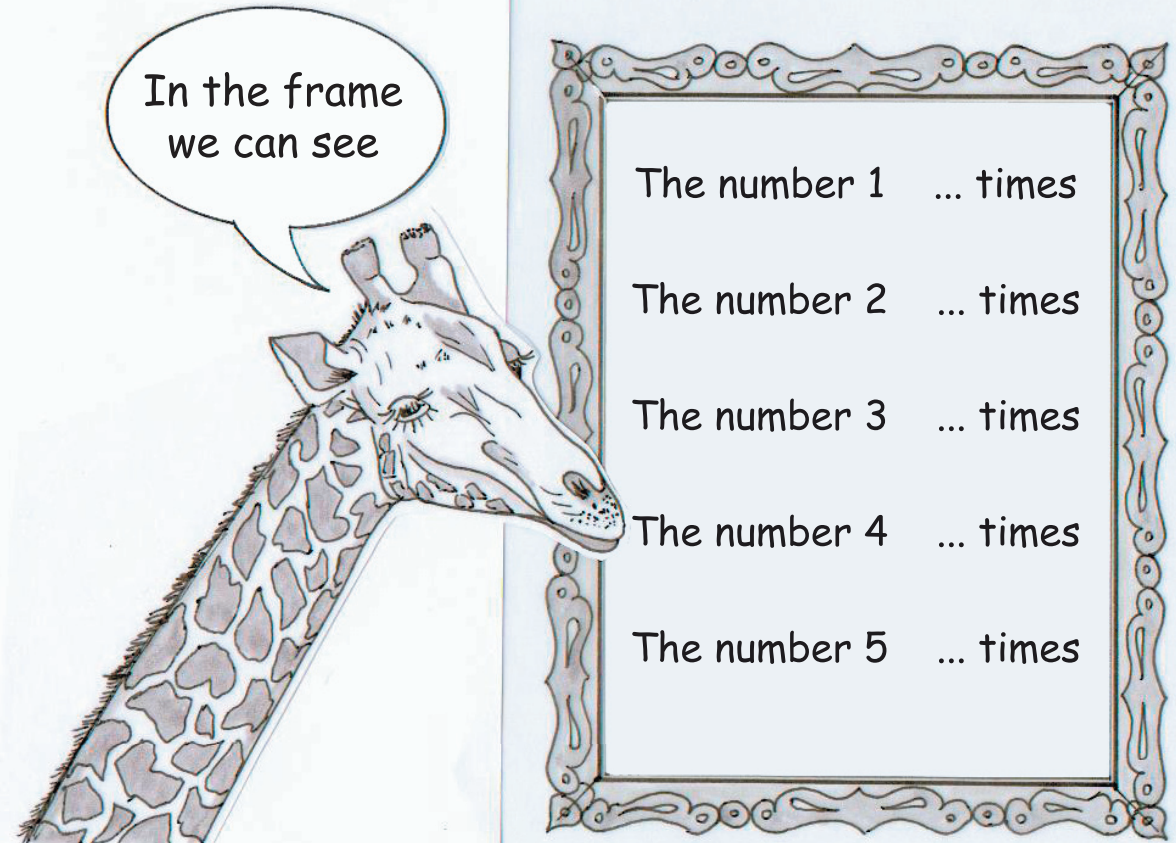

And here we have an octahedron O_2 which has side 2 units and is made from several tetrahedrons T_1 and octahedrons O_1. The visible faces of the octahedron are shown in grey.

How many tetrahedrons T_1 and octahedrons O_1 do you need to construct a tetrahedron T_4 which has side 4 units ? And how many to make an octahedron of side 4 ? Explain your answer.

Fill in the numbers on the table so that all the statements are true.

In the frame
we can see

The number 1   ... times

The number 2   ... times

The number 3   ... times
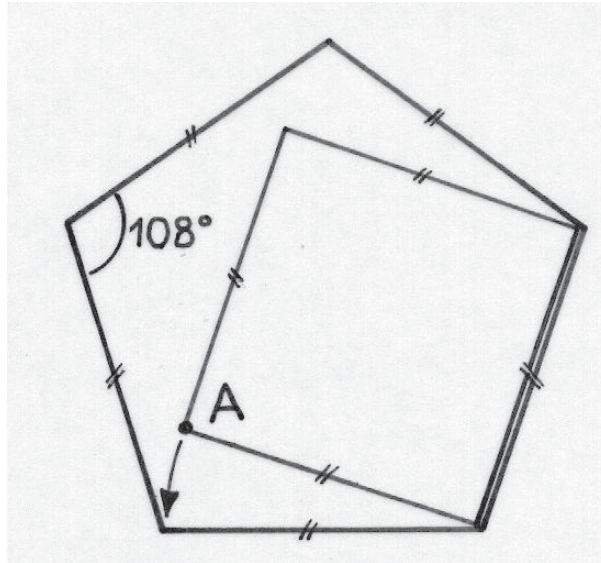
The number 4   ... times

The number 5   ... times

A square of side 8 cm rolls round a regular pentagon which also has side 8 cm. One of the vertices of the square is always touching a vertex of the pentagon.

Draw on your answer sheet in red the path traced out by the vertex A of the square as it moves round.

Question 8 Without breaking eggs

Maria is going to put the smallest possible number of eggs into a basket so that

if you take out the eggs in twos there will be one left ;
if you take them out in threes there will be 2 left ;
if you take them out in fours there will be 3 left ;
if you take them out in fives there will be 4 left ;
if you take them out in sixes there will be 5 left ;
and if you take them out in sevens there will be none left.

How many eggs should she put in the basket ? Justify your answer.

Question 9 Card play

Little Joe and Old Firehand are going to play cards with the famous gambler Black Jacky. They are using a pack of 32 cards numbered 1 to 32.
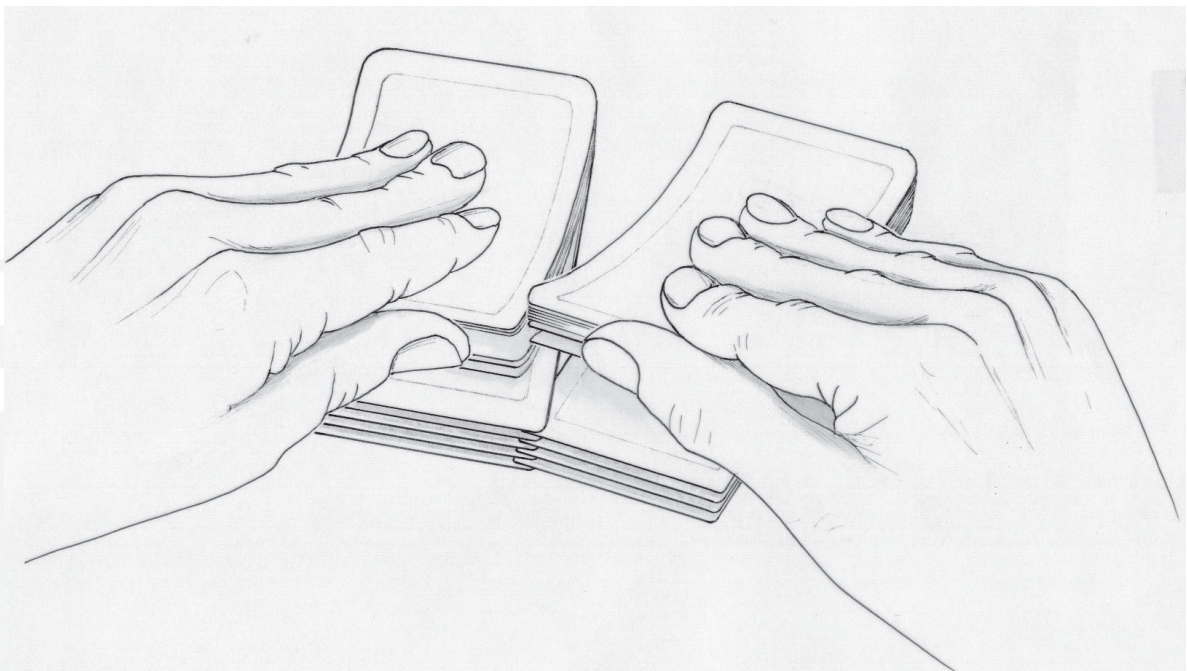
Having explained the rules of the game Black Jacky shuffles the cards.

To do this he places the pack on the table, takes exactly 16 cards from the top and places them to the right of the pack. He does not turn them over.
He mixes them up by taking cards alternately from each pile. He starts by putting the card from the bottom of the left hand pile on the table. He shuffles the pack in this way and then does the same process again several times.

Little Joe does not think this is a good way to shuffle cards.
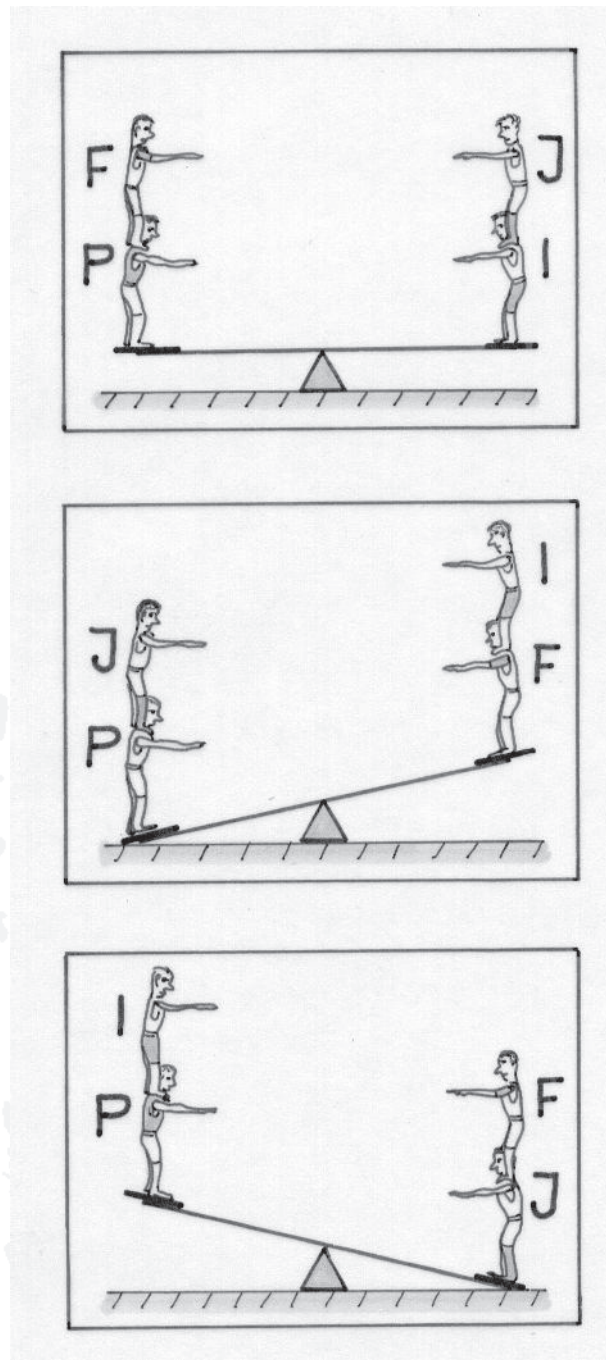Show that after a few shuffles of this kind you get a very surprising result.

Question 10 Balancing act

Here are 3 pictures of brothers Paul, Jean, Igor and Franck on a see-saw.

Who is the heaviest ? Who is the lightest ?
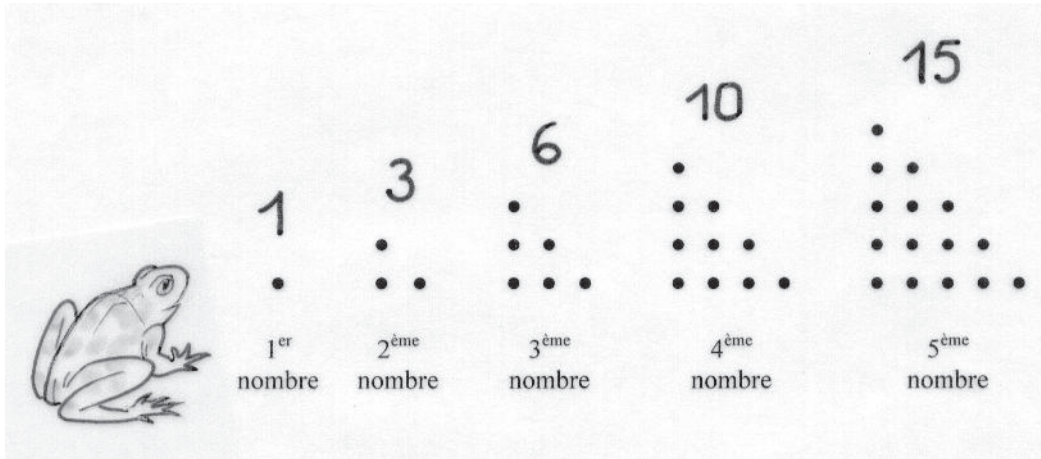Can you put the four brothers in order of weight ? Justify your answer.
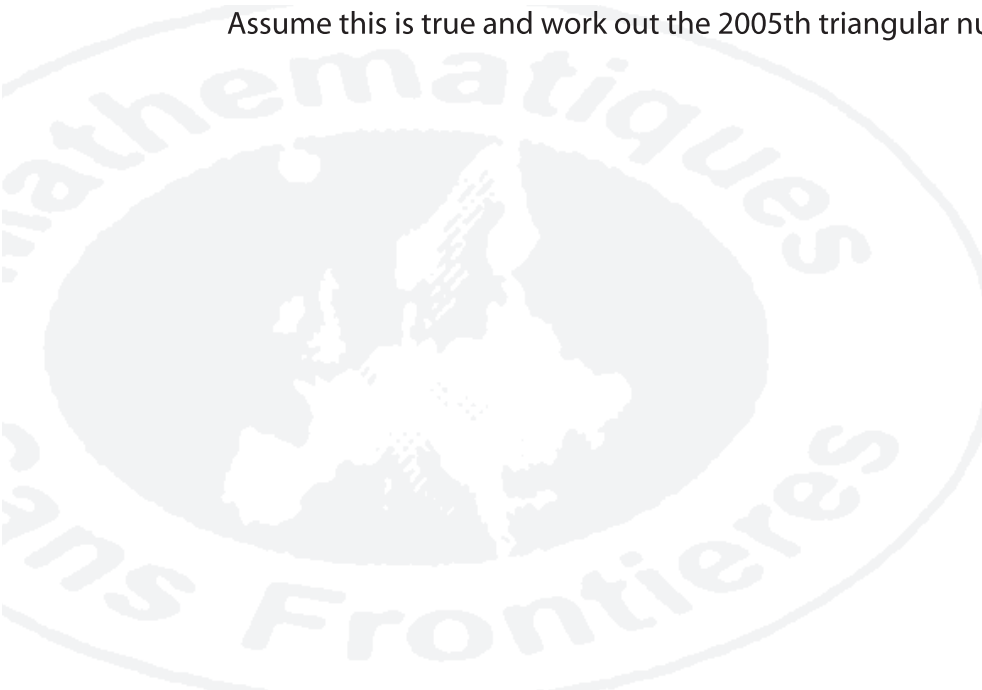
Question 11 Triangles

Senior classes only

The diagram below shows the first 5 triangular numbers.

Show in a diagram or by calculation that the sum of two consecutive triangular numbers is a square number. Use at least three examples.

Assume this is true and work out the 2005th triangular number. Show your working.

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Question 12 On your bike

Senior classes only

Two cyclists Paulette and Yves meet up on the road. When they meet their speedometers are showing an average speed of 24 km/h for Paulette and 30 km/h for Yves. They cycle together for an hour and cover 27 km before they part.

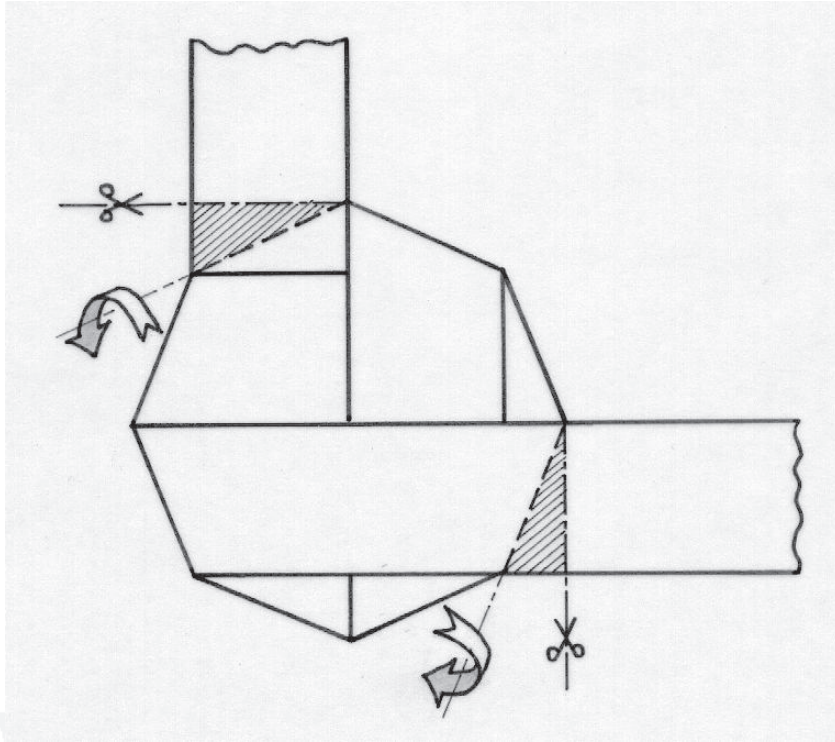As they part Paulette's speedometer shows an average speed of 25 km/h and Yves's shows 29 km/h.

What is the total distance Paulette has covered. And how far has Yves cycled ?

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Question 13 Knotty problem

Yolande wraps a ribbon round a regular octagon which could be inscribed in a circle of radius 4 cm. Look at the diagram below.

The ribbon as it is wrapped round covers both faces of the octagon exactly.

Calculate the width of the ribbon and the minimum length it would need to be to cover the two faces of the octagon completely.
Make a model and stick it to your answer sheet.